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APPLICATION OF SLENDER BODY THEORY IN SHIP HYDRODYNAMICS AT HIGH-FROUDE NUMBER

C.J. Garrison

May 1977

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The slender body theory utilizing the method of inner and outer expansions is applied to evaluate the yawing force and moment on a slender ship hull at high-Froude number. The theory is developed for hulls of arbitrary cross-section, and a Green's function method is outlined for solving the inner problem for hulls of arbitrary section. The theory is applied to a slender cylinderical hull, and the potential is represented in the inner region by a multipole expansion. Closed form results are obtained for the yawing force and moment for the cylindrical hull.

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1. Introduction

Slender body theory has enjoyed great success in aerodynamics, and in recent years there has been considerable interest in applications in naval hydrodynamics. Ship hulls are long and slender, and this is a logical feature to exploit in simplification of the governing equations. Michell's classical theory of wave-making resistance published in 1898 considered the ship to be a "thin" body and represented the hull by use a of distribution of "Havelock sources" over a vertical mid-plane. However, ships tend to be "slender" as opposed to "thin" because the beam and draft tend to be of the same order of magnitude and both tend to be small in comparison to the length. It was hoped, therefore, that slender body theory might represent a significantly better alternative to Michell's theory in the area of wave resistance and at the same time yield useful results for the hydrodynamic forces in the transverse direction both in steady and unsteady motion. Newman (1970) has given a review of the application of slender body theory to ship hulls which discusses various kinds of hull motion and wave resistance.

Major advances in the application of slender body theory in ship hydrodynamics have been made largely in the past decade or so principally by Tuck (1963,1964), Newman (1964,1965), Ogilvie (1969), Ursell(1968) and Faltinsen(1971). The method of approach involves the use of matched asymptotic expansions which is a powerful method for dealing with singular perturbation problems. Potential flow past a slender body is recognized as a singular perturbation problem and in such cases an inner and outer expansion is developed. The inner expansion is valid in the near-field, i.e., near the body and the outer expansion is valid in the far-field. The unknowns remaining in either of the expansion are determined by a matching procedure. The general method of matched asymptotic expansions is discussed by Van Dyke(1964).

In the present report the application of the slender body theory to a ship hull in steady motion at high-Froude number is considered. While most displacement ships operate in the zero to intermediate Froude number range, there is a new generation of high speed ships generally of the catamaran configuration which utilize rather thin or slender hulls and operate at rather high Froude number. The surface effects ship hulls represent the principal example of this type of hull even though the flow about an SES hull is somewhat more complex than the one considered here because of the pressure difference between the two sides of the hull. In the following a single slender hull operating a high-Froude number is considered from the view point of slender body theory.

2. Boundary-value Problem For Ship Hydrodynamics

We assume the fluid to be ideal (inviscid, incompressible and homogeneous) and the motion to be irrotational. If (x,y,z) represents Cartesian coordinates fixed in space with the gravitational force acting in the negative y-direction and t denotes time, a velocity potential $\hat{\phi}(x,y,z,t)$ can be introduced such that the fluid velocity vector is given by

$$\bar{q} = \nabla\hat{\phi} \tag{1}$$

The condition of incompressible flow applied to (1), i.e., $\nabla . \bar{q} = 0$ yields Laplace's equation as the governing differential equation,

$$\nabla^2\hat{\phi} = \hat{\phi}_{xx} + \hat{\phi}_{yy} + \hat{\phi}_{zz} = 0 \tag{2}$$

The fluid pressure $P(x,y,z,t)$ is expressed in terms of $\hat{\phi}$ and the elevation y, as

$$P = -\rho\hat{\phi}_t - \tfrac{1}{2}\rho\nabla\hat{\phi}.\nabla\hat{\phi} - \rho g y + C(t) \tag{3}$$

where $C(t)$ may be, in general, a function of time, ρ denotes the mass density of the fluid, and g, the acceleration of gravity. The function $C(t)$ is at most a function of time, and can be determined by evaluation of $P+\rho\hat{\phi}_t+\frac{1}{2}\rho\nabla\hat{\phi}\cdot\nabla\hat{\phi}+\rho gy$ at some reference point. This point is generally taken at the free surface at infinity where all of the values are known.

The surface of the hull is described by the equation $s(x,y,z,t)=0$. Since this surface is considered to be impermeable the kinematic boundary condition, which imposes the condition that the relative fluid velocity in a direction normal to the hull must vanish, is given by

$$\frac{Ds}{Dt} = \frac{\partial s}{\partial t} + \nabla\hat{\phi}\cdot\nabla s=0 \quad \text{on } s=0 \tag{4}$$

On the free surface there are two boundary conditions which must be applied. If the elevation of the free surface is described by $y=\eta(x,z,t)$ the kinematic boundary condition obtained in a fashion similar to (4) results from setting the substantial deriative of the surface $\eta-y=0$ to zero,

$$\frac{D}{Dt}(\eta-y)=\eta_t+\hat{\phi}_x\eta_x+\hat{\phi}_z\eta_z-\hat{\phi}_y = 0 \quad \text{on } y=\eta \tag{5}$$

In addition to (5) a dynamic boundary condition must also be imposed. This boundary condition results from simply applying $P=0$ on the free surface and is given by

$$\hat{\phi}_t + \tfrac{1}{2}\nabla\hat{\phi}\cdot\nabla\hat{\phi}+gy=C(t) \quad \text{on } y=\eta \tag{6}$$

where $C(t)$ is evaluated with the aid of a known reference point in the fluid.

In addition to the boundary conditions (4-6) the potential function must satisfy certain radiation conditions at $x^2+y^2 \rightarrow \infty$ appropriate to the particular problem under consideration. Also, on the bottom the kinematic boundary condition

$$\hat{\phi}_y = 0 \quad \text{on } y=-h \tag{7}$$

is applied if the water depth is finite and denoted by h.

3. Perturbation Procedure For a Slender Body

At this juncture let us consider a particular problem: a slender hull is positioned with its long axis coincident with the x-axis and held fixed in space as indicated in Figure 1. A uniform stream of

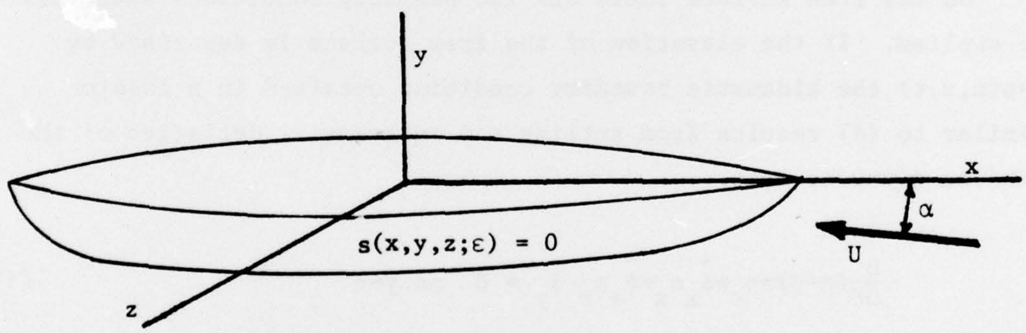

Figure 1 Definitions

magnitude U makes an angle α with respect, to the x-axis so that the resulting x,y, and z components are $-U \cos \alpha$, 0, $-U \sin \alpha$, respectively. The coordinate system is placed at the mean free surface so that the y=0 plane represents the mean free surface,and the water depth is taken as infinite. The motion will be restricted to steady motion,and the

hull is described by $s(x,y,z;\varepsilon) = 0$ where the parameter ε = beam or draft-to-length ratio.

Outer Problem

The problem which is here defined is recognized to be dependent on two dimensionless parameters, the beam/length ratio, ε, and the incidence angle, α. Thus, as an initial step we may expect the solution to be expandable in a power series in both ε and α. (At least the solution should behave in a regular manner with the exception of the region near the body.) Introducing the body potential, ϕ, the total potential may be expressed as

$$\hat{\phi} = \phi(x,y,z;\varepsilon,\alpha) - Ux \cos \alpha - Uz \sin \alpha \tag{8}$$

where the body potential $\bar{\phi}$ is expressed by

$$\phi = \varepsilon\phi_1^{\varepsilon} + \alpha\phi_1^{\alpha} + O(\varepsilon^2) + O(\alpha^2) \tag{9}$$

in the outer region. The first term of the Taylor series, $\phi(0,0) = \phi_0$, is, of course, zero because if both $\varepsilon=0$ and $\alpha=0$ the uniform stream is directed parallel to a line of zero thickness and the body potential vanishes. Thus, expanding $\sin\alpha$ and $\cos\alpha$ about $\alpha=0$ in (8) and substituting into (9) gives the total potential as

$$\hat{\phi} = \varepsilon\phi_1^{\varepsilon} + \alpha(\phi_1^{\alpha} - Uz) - Ux + O(\varepsilon^2) + O(\alpha^2) \tag{10}$$

and substituting this form of the potential into (2) shows that

$$\nabla^2\phi_1^{\varepsilon} = \nabla^2\phi_1^{\alpha} = 0 \tag{11}$$

since α and ε are independent parameters.

In similar fashion,we may express the free surface elevation in the form of (9),

$$\eta=\epsilon\eta_1^{\epsilon}+\alpha\eta_1^{\alpha}+O(\epsilon^2)+O(\alpha^2)+O(\alpha\epsilon) \tag{12}$$

Substituting (8),(9) and (12) into (5) and collecting coefficients of like powers of ϵ and α gives:

$$\phi_{1y}^{\epsilon}+U\eta_{1x}^{\epsilon}=0 \quad \text{on } y=0 \tag{13}$$

and

$$\phi_{1y}^{\alpha}+U\eta_{1x}^{\alpha}=0 \quad \text{on } y=0 \tag{14}$$

The dynamic free surface boundary condition is obtained by use of (6) by evaluating $C(t)=\frac{1}{2}U^2$ at $x=\infty$ on $y=0$ and substituting in the form of $\hat{\phi}$ and η as given in (12) and (8). Here again two separate boundary conditions result because of the independence of the two small parameters, ϵ and α. The first of these is obtained by setting $\alpha=0$ as

$$\eta_1^{\epsilon}+\frac{U}{g}\phi_{1x}^{\epsilon}=0 \quad \text{on } y=0 \tag{15}$$

and the second by setting $\epsilon=0$,

$$\eta_1^{\alpha}+\frac{U}{g}\phi_{1x}^{\alpha}=0 \quad \text{on } y=0 \tag{16}$$

Eqs. (13) and (14) together with (15) and (16) yield the linear free surface boundary conditions,

$$\phi_{1y}^{\epsilon}-\frac{U^2}{g}\phi_{1xx}^{\epsilon}=0 \tag{17}$$

and

$$\phi_{1y}^{\alpha}-\frac{U^2}{g}\phi_{1xx}^{\alpha}=0 \tag{18}$$

Eqs. (17) and (18) represent familiar linear boundary conditions for the free surface. If we now let the Froude number U^2/gL (where L is considered to be O(1)) become large, (17) and (18) become simply

$$\phi_1^{\varepsilon} = 0 \qquad \text{on } y=0 \tag{19}$$

$$\phi_1^{\alpha} = 0 \qquad \text{on } y=0 \tag{20}$$

In summary, we have derived a boundary-value problem for the outer potential. The solution is singular in the region of the body so the hull kinematic boundary condition is not applied to the outer solution. We, therefore, have in summary for ϕ_1^{ε},

$$\phi_{1_{xx}}^{\varepsilon} + \phi_{1_{yy}}^{\varepsilon} + \phi_{1_{zz}}^{\varepsilon} = 0 \tag{22}$$

$$\phi_1^{\varepsilon} = 0 \quad \text{on } y=0 \tag{23}$$

and similarly for ϕ_1^{α},

$$\phi_{1_{xx}}^{\alpha} + \phi_{1_{yy}}^{\alpha} + \phi_{1_{zz}}^{\alpha} = 0 \tag{24}$$

$$\phi_1^{\alpha} = 0 \quad \text{on } y=0 \tag{25}$$

The boundary condition on the hull is in this case to be replaced by a matching condition.

Inner Problem

We now consider the inner potential which is valid near the slender body. The solution to the slender body problem is singular in the inner region but the degree of singularness at the outset is unknown. We there-

fore begin by writing the body potential as

$$\phi = \Delta_o(\varepsilon)\Phi_o(x,Y,Z;\alpha)+\Delta_1(\varepsilon)\Phi_1(x,Y,Z,;\alpha)+\ldots \tag{26}$$

where the Y and Z coordinates are stretched by the parameter ε,

$$Y=y/\varepsilon \tag{27}$$

$$Z=z/\varepsilon \tag{28}$$

and $\Delta_o(\varepsilon),\Delta_1(\varepsilon)$, etc. are called gage functions as defined by Van Dyke (1964). These gage functions will eventually be determined by use of the principle of least degeneracy also discussed by Van Dyke.

The body potential is expressed in (26) as an inner perturbation series in ε, the beam-length ratio. Here the incidence angle, α, is allowed to remain finite and is not considered in any limiting process. We shall return to this point later.

If the inner representation of the body potential given by (26) is substituted into (8) and the result substituted into the Laplacian, (2), we find,

$$\varepsilon^2\Phi_{o_{xx}}+\Phi_{o_{YY}}+\Phi_{o_{ZZ}}+\frac{\Delta_1(\varepsilon)}{\Delta_2(\varepsilon)}\left[\varepsilon^2\Phi_{1_{xx}}+\Phi_{1_{YY}}+\Phi_{1_{ZZ}}\right]+\ldots=0 \tag{29}$$

Clearly, if $\Delta_1(\varepsilon)/\Delta_2(\varepsilon)$ is at least of $O(\varepsilon)$ then the three dimensional Laplacian degenerates to the two-dimensional form for the leading term in the inner expansion:

$$\Phi_{o_{YY}}+\Phi_{o_{ZZ}}=0 \tag{30}$$

The inner representation, being valid near the hull, must satisfy the boundary condition on the hull. If s is expressed in inner variables as

$s(x,y,z;\varepsilon) = S(x,Y,Z;\varepsilon) = 0$ then the hull kinematic boundary condition, which is obtained by use of (26) and (8) with (4), gives

$$\left(\varepsilon^2 \Phi_{o_x} S_x + \Phi_{oY} S_Y + \Phi_{oZ} S_Z\right) + \frac{\Delta_1(\varepsilon)}{\Delta_o(\varepsilon)} \left(\varepsilon^2 \Phi_{1_x} S_x + \Phi_{1_Y} S_Y + \Phi_{1_Z} S_Z\right) \tag{31}$$

$$+ \ldots\ldots \quad - \frac{\varepsilon^2 U S_x}{\Delta_o(\varepsilon)} - \frac{\varepsilon U \alpha S_Z}{\Delta_o(\varepsilon)} = 0$$

Now if α were to be considered to be fixed at $O(1)$ then the principle of least degeneracy would require that $\Delta_o(\varepsilon) = \varepsilon$, and the hull kinematic condition would read

$$\Phi_{oY} S_Y + \Phi_{oZ} S_Z = U\alpha S_Z \quad \text{on } S=0 \tag{32}$$

This boundary condition simply indicates that in any transverse plane perpendicular to the x-axis the body potential must just cancel the cross-flow velocity, $U\alpha$. It is noted that in such a case the cross-flow is an order of magnitude larger than the perturbation associated with the thickness distribution of the body and thickness effect is, therefore, lost.

Alternately, if the angle of attack were taken to be zero then the principle of least degeneracy leads one to $\Delta_o(\varepsilon)=\varepsilon^2$ and (31) gives,

$$\Phi_{oY} S_Y + \Phi_{oZ} S_Z = U S_x \quad \text{on } S=0 \tag{33}$$

a familiar result in slender body theory.

A third possibility is to take α to be of the same order of magni-

tude as ε and retain both of the terms on the right hand side of (32) and (33). This gives $\Delta_o(\varepsilon)=\varepsilon^2$ and

$$\Phi_{o_Y} S_Y + \Phi_{o_Z} S_Z = U\left(S_x + \frac{\alpha}{\varepsilon} S_Z\right) \text{ on } S=0 \tag{34}$$

An alternate way of writing (34) is

$$\frac{\partial \Phi_o}{\partial N} = U\left(\frac{S_x}{S_N} + \frac{\alpha}{\varepsilon}\frac{\partial N}{\partial Z}\right) \tag{35}$$

where $N = n/\varepsilon$ and n represents distance measured normal to the curve in the Y-Z plane described by the equation $S(x,Y,Z;\varepsilon) = 0$ with x held fixed.

The free surface condition to be applied to the inner solution is obtained by eliminating η between (5) and (6). The boundary condition obtained in this way is

$$\hat{\phi}_x(\nabla\hat{\phi}\cdot\nabla\hat{\phi}_x)+\hat{\phi}_z(\nabla\hat{\phi}\cdot\nabla\hat{\phi}_z)+g\hat{\phi}_y=0 \quad \text{on } Y=\eta/\varepsilon \tag{36}$$

Substitution of (26) and (8) into (36) gives

$$\begin{aligned}&(\varepsilon^2\Phi_{o_x}-U)\left[\varepsilon^2\Phi_{o_{xx}}(\varepsilon^2\Phi_{o_x}-U)+\varepsilon^2\Phi_{o_Y}\Phi_{o_{xY}}+\varepsilon\Phi_{o_{xZ}}(\varepsilon\Phi_{o_Z}-U\alpha)\right.\\ &+(\varepsilon\Phi_{oZ}-U\alpha)\left[\varepsilon\Phi_{o_{xZ}}(\varepsilon^2\Phi_{ox}-U)+\varepsilon\Phi_{oY}\Phi_{oYZ}+\Phi_{oZZ}(\varepsilon\Phi_{oZ}-U\alpha)+g\varepsilon\Phi_{o_Y}=0 \text{ on } Y=0\right.\end{aligned} \tag{37}$$

Now, if $\varepsilon\to 0$ with α=fixed, (37) gives

$$\Phi_{oZZ}=0 \quad \text{on } Y=0 \tag{38}$$

or from this we can deduce that

$$\Phi_o=0 \quad \text{on } Y=0 \tag{39}$$

is the appropriate boundary condition in the inner region. The problem for the first term of the inner expansion for the high-Froude number

slender body theory is, in summary,

$$\Phi_{oYY}+\Phi_{oZZ}=0 \tag{40a}$$

$$\frac{\partial\Phi_o}{\partial N}=U\left(\frac{S_x}{S_N}+\frac{\alpha}{\varepsilon}\frac{\partial N}{\partial Z}\right) \text{ on } S=0 \tag{40b}$$

$$\Phi_o=0 \quad \text{on } Y=0 \tag{40c}$$

There is no radiation condition specified in (40); this condition is replaced by matching with the outer solution.

4. Solution for the Inner Problem

The inner problem specified in (40) is essentially a two-dimensional problem as indicated in Figure 2. Such problems can be solved for rather

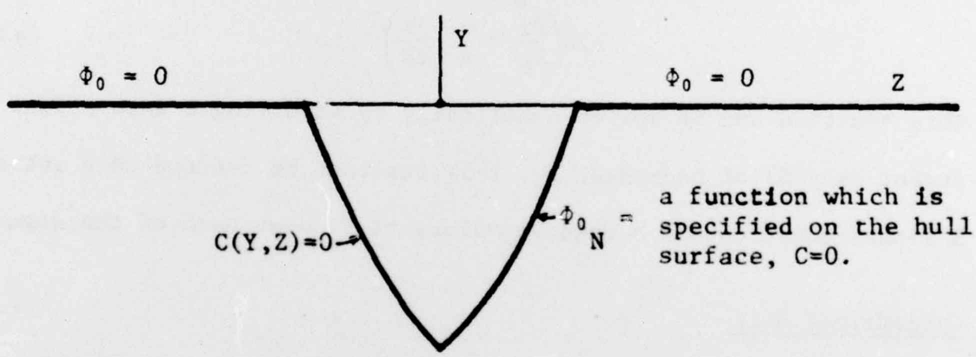

Figure 2 Inner Problem

arbitrary shapes by use of a Green's function of the form

$$G(Z,Y;\zeta,\eta)=\ln\sqrt{(Z-\zeta)^2+(Y-\eta)^2}-\ln\sqrt{(Z-\zeta)^2+(Y+\eta)^2} \tag{41}$$

The first term in (41) represents a source, and the second term represents a negative image with respect to the Z axis. The potential Φ_o may be represented by a surface distribution over the hull of these two-dimensional singularities.

In particular, the potential may be represented by

$$\Phi_o = \frac{1}{2\pi} \int_C F(x;\eta,\zeta) G(Z,Y;\zeta,\eta)\, dC \qquad (42)$$

where C denotes the curve in the Y-Z plane representing the cross-section of the hull and dC denotes an arc length along that curve. Using (41) for G, (40a) and (40c) are automatically satisfied. The hull boundary condition (40b) then yields the integral equation for the source strength distribution, $F(x;Y,Z)$ as

$$F(x;Y,Z) + \frac{1}{\pi} \int_C F(x;\eta,\zeta) \frac{\partial G}{\partial N} (Z,Y;\zeta,\eta)\, dC$$
$$= 2U\left(\frac{S_x}{S_N} + \frac{\alpha}{\varepsilon} \frac{\partial N}{\partial Z}\right) \quad \text{on } C \qquad (43)$$

This equation can be solved numerically by subdiving C into a finite number (say N) of segments. Eq. (43) can then be reduced to a set of N linear equations in N unknown values of F_i over each of the segments.

Cylindrical Hull

For the case of a semi-immersed circular cylinder, a multipole series may be used to represent the solution. Consider the surface described in cylindrical coordinates by

$$S(R,\theta,x) = R - R_H(x) \qquad (44)$$

where $R = r/\varepsilon$ denotes the radial coordinate in inner variables and R_H

denotes the radius of the hull in the inner variable ($R_H = r_H/\varepsilon$) and is, in general, a function of x. The kinematic boundary condition on the hull, (40b), takes the form,

$$\frac{\partial \Phi_o}{\partial R} = U(R_H' + \frac{\alpha}{\varepsilon}\cos\theta) \quad \text{on } R=R_H \tag{45}$$

where the polar coordinate system is related to the cartesian system by

$$Z=R\cos\theta \tag{46a}$$

$$Y=-R\sin\theta \tag{46b}$$

The solution for Φ_o satisfying both (40a) and (40b) is given by

$$\Phi_o=\sum_{n=1}^{\infty} \frac{a_n \sin(n\theta)}{R^n} \tag{47}$$

Application of the hull boundary condition (45) now gives

$$\sum_{n=1}^{\infty} \frac{a_n n}{R_H^{(n+1)}} \sin n\theta=-UR_H' - \frac{\alpha}{\varepsilon} U\cos\theta \tag{48}$$

By use of the results

$$\int_0^\pi \sin^2(n\theta)d\theta=\int_0^\pi \cos^2(n\theta)d\theta=\pi/2 \tag{49}$$

$$\int_0^\pi \sin(n\theta)\sin(m\theta)=\int_0^\pi \cos(n\theta)\cos(m\theta)d\theta=0 \quad \text{if } m\neq n \tag{50}$$

$$\int_0^\pi \sin(n\theta)\cos\theta\, d\theta= \frac{2n}{n^2-1}, \quad n\neq 1 \tag{51}$$

$$\int_0^\pi \sin(n\theta)d\theta= \frac{1-\cos(n\pi)}{n} = \frac{1-(-1)^n}{n} \tag{52}$$

we find

$$a_n = -\frac{\alpha}{\varepsilon}\ \frac{UR_H^{(n+1)}}{n}\ \frac{2}{\pi}\int_0^{\pi} \cos\theta\ \sin(n\theta)\, d\theta, \quad n=2,4,6,\ldots. \tag{53}$$

or, carrying out the integration,

$$a_n = -\frac{\alpha}{\varepsilon}\ \frac{4}{\pi}\ UR_H^{(n+1)} \left(\frac{1}{n^2-1}\right), \quad n=2,4,6,\ldots. \tag{54}$$

and

$$a_n = -\ \frac{2\ UR_H'\ R_H^{(n+1)}}{\pi} \left(\frac{1-(-1)^n}{n^2}\right), \quad n=1,3,5,\ldots \tag{55}$$

The inner potential function is, therefore, represented by the series

$$\begin{aligned} \Phi_o = &- \frac{\alpha}{\varepsilon}\ \frac{4U}{\pi} \sum_{n=2,4}^{\infty} \left(\frac{1}{n^2-1}\right) \frac{R_H^{(n+1)}}{R^n} \sin(n\theta) \\ &- \frac{2}{\pi}\ UR_H' \sum_{n=1,3,..}^{\infty} \left(\frac{1-(-1)^n}{n^2}\right) \frac{R_H^{n+1}}{R^n} \sin(n\theta) \end{aligned} \tag{56}$$

The dynamic pressure on the hull can be obtained from Bernoulli's equation, disregarding terms of $O(\varepsilon^4)$ and $O(\varepsilon^2\alpha^2)$,

$$P = \rho\varepsilon^2 \left\{ U\Phi_{o_x} - \left(\Phi_{o_R}^2 + \frac{1}{R^2}\ \Phi_{o_\theta}^{2}\right) - \left(\frac{\alpha}{\varepsilon}\right)\frac{2U}{R}\Phi_{o_\theta}\sin\theta + \left(\frac{\alpha}{\varepsilon}\right)^2 U^2 \sin^2\theta \right\} \tag{57}$$

The normal (or yaw) force per unit length along the hull can be determined through integration of the pressure over the immersed surface. That is,

$$dF_z = \left[-\int_0^{\pi} P\ R_H \cos\theta\ d\theta\right] dx \tag{58}$$

After substitution of (56) into (58) and some lengthy manipulations and integrations we get, for the normal force on the segment dx,

$$dF_z = -\varepsilon^2 \alpha \rho U^2 \frac{4}{\pi} R_H R_H' \left\{ 2 + \sum_{n=2}^{\infty} \left(\frac{1-(-1)^n}{n} - \frac{n(1+(-1)^n}{n^2-1} \right) \right\} dx \quad (59)$$

The net normal force is obtained by integration from the bow where $R_H=0$ to the stern where $R_H=R_{H_S}$,

$$F_z = -\alpha \frac{1}{2} \rho U^2 \frac{\pi}{2} r^2_{H_S} \frac{8}{\pi^2} \left\{ 2 + \sum_{n=2}^{\infty} \left(\frac{1-(-1)^n}{n} - \frac{n(1+(-1)^n)}{n^2-1} \right) \right\} \quad (60)$$

The dimensionless normal (or yaw) force coefficient based on the stern area, therefore, becomes

$$f_z = \frac{-F_z}{\frac{1}{2}\rho U^2 \frac{\pi}{2} r^2_{H_S}} = \frac{8}{\pi^2} \left\{ 2 + \sum_{n=2}^{\infty} \left(\frac{1-(-1)^n}{n} - \frac{n(1+(-1)^n)}{n^2-1} \right) \right\} \alpha \quad (61)$$

where r_{H_S} is defined as $R_{H_S}=r_{H_S}/\varepsilon$ and r_H represents the hull radius in outer variables. For boat-tailed hulls we get the familiar result $F_z=0$ because $R_{H_S}=0$.

The summation in (61) can be somewhat rearranged to read

$$f_z = \frac{8}{\pi^2} \left\{ \frac{2}{3} + 2 \sum_{n=3,5,7}^{\infty} \frac{1}{n(n+2)} \right\} \alpha \quad (62)$$

The series converges rapidly to approximately 0.128 so

$$f_z = 0.75\alpha \quad (63)$$

The value of $f_z=0.75\alpha$ for the cylindrical hull at high Froude number can be compared with the classical value of $f_z=2\alpha$ for the same hull operating

with the low-speed free surface boundary condition, $\partial\Phi_o/\partial Y=0$ on $Y=0$. It is reasonable to expect a somewhat smaller value for f_z for the high-Froude number case because the high-Froude number free surface boundary condition makes the Y=0 plane act like a zero pressure surface and, therefore, right at the free surface there can be no pressure difference across the hull. The low Froude number(actually low to moderate) boundary condition makes the free surface act like a rigid wall and a pressure difference across the hull can exist at the Y=0 level.

The moment about the Y-axis is given by

$$dM_y = -\int_{x=-a}^{b} x\, dF_z \tag{64}$$

where a and b denote the distance from the origin to the stern and bow, respectively, so that the hull length is L=a+b. Thus, using (59) with (64) gives

$$m_y = \frac{M_y}{\frac{1}{2}\rho U^2 \forall} = \frac{8}{\pi^2}\left(1-\frac{A_s^2 L}{\forall}\right)\left\{2+\sum_{n=2,3,..}\left(\frac{1-(-1)^n}{n}-\frac{n(1+(-1)^n)}{n^2-1}\right)\right\}\alpha \tag{65}$$

where

A_s=area of the stern section below the mean waterline

$\forall$=hull displacement

L=hull length

When the series is evaluated we get

$$m_y = 0.75\left(1-\frac{A_s^2\, L}{\forall}\right)\alpha \tag{66}$$

and for boat-tailed (or closed) bodies the moment coefficient becomes $m_y = 0.75\alpha$. This value may be contrasted with the low Froude number result of 2.0α which is often referred to as Munk's moment.

5. Conclusions

Slender body theory has been applied to a slender ship hull operating at high-Froude number. For the case of a semi-circular hull, closed-form results were obtained for the yaw force and moment. These values are less than half the values of the equivalent coefficients associated with the rigid-wall free surface boundary condition.

REFERENCES

Faltinsen, O.M.(1971)"Wave Forces on a Restrained Ship in Head-Sea Waves", Ph.D. thesis, The University of Michigan (Also: A Rational Strip Theory of Ship Motions: Part II, Department No. 113, Department of Naval Architecture & Marine Engineering The University of Michigan, Ann Arbor, Michigan.)

Newman, J.N., (1970) "Applications of Slender-Body Theory in Ship Hydrodynamics", Annual Reviews of Fluid Mechanics, Vol.2.

Newman,J.N.,(1965) David Taylor Model Basin Report. No. 2127

Newman,J.N.,(1964) "A Slender-body Theory for Ship Oscillation in Waves", Journal of Fluid Mechanics, Vol.18 pp 602-608.

Newman,J.N. and Tuck,E.O.,(1964) "Current Progress in the Slender-Body Theory for Ship Motion," Procurement Proceedings Fifth Symposium on Naval Hydrodynamics, Bergen, Norway ,(1964)(U.S. Government Printing Office, Washington, D.C.) pp 129-162.

Ogilvie,T.F. and Tuck,E.O.,(1969)"A Rational Strip Theory of Ship Motions", University of Michigan, Department of Naval Architecture and Marine Engineering Dept. No. 013.

Tuck,E.O.,(1964)"A Systematic Asumptotic Expansion Procedure for Slender Ships", Journal of Ship Research, Vol.8, No.1, pp 15-23.

Tuck,E.O.,(1964), "On Line Distributions of Kelvin Sources" Journal of Ship Research, Vol.8 No.2, pp 45-52.

Tuck,E.O.,(1963)"Steady Motion of a Slender Ship",(Doctoral Thesis, University of Cambridge.)

Ursell,F.(1968),"On Head Seas Travelling along a Horizontal Cylinder", J. Inst. Math. Applications, Vol. 4 (414-427)

Van Dyke,M.(1964),"Perturbation Methods in Fluid Mechanics"(Academic Press, New York and London, 229 p.p.)

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